Theory –

Problem Formulation-

Objective Function: We assume each generator has a cost function (typically having quadratic, linear and no-load cost coefficients):

- Quadratic cost function -

$$C_i(P_i) = a_i P_i^2 + b_i P_i + c_i$$

  where:
  P_i = power output of generator i,
  a_i = Quadratic cost coefficient of generator i.
  b_i = Linear cost coefficient of generator i.
  c_i = No-load cost coefficient of generator i

- Linear cost function –

$$C_i(P_i) = b_i P_i + c_i$$

- Incremental cost –

$$\frac{dC_i}{dP_i} = \begin{cases} 2a_i P_i + b_i & \text{For quadratic cost function} \\ b_i & \text{For linear cost function} \end{cases}$$

- Total generation cost to minimize:

$$C_{Total} = \sum_{i=1}^{N} C_i(P_i)$$

Constraints

1. Power balance constraint (equality constraint):

$$\sum_{i=1}^{N} P_i = P_D$$

   Where, P_D = total system demand and N is the number of generators.
2. Generator limits (inequality constraints):

$$P_i^{min} \leq P_i \leq P_i^{max}, \qquad i = 1,2,\dots,N$$

Solution: Using Classical Method (Lagrangian Multiplier)

$$\mathcal{L} = \sum_{i=1}^{N} C_i(P_i) + \lambda(P_D - \sum_{i=1}^{N} P_i)$$

By taking the derivative with P_i:

$$\frac{\partial L_i}{\partial P_i} = \frac{\partial C_i}{\partial P_i} - \lambda = 0 => \frac{\partial C_i}{\partial P_i} = \lambda$$

For generator with linear cost function the generator with lowest incremental cost (λ) value will be scheduled first. Moreover, in case of the generator with quadratic cost function, the generator schedule will be decided as per the optimality condition given below. At optimal dispatch, the incremental cost (λ) of all generators must be equal.

$$\frac{\partial C_1}{\partial P_1} = \frac{\partial C_2}{\partial P_2} = \dots = \frac{\partial C_N}{\partial P_N} = \lambda$$

Handling the inequality constraint (λ iteration method)

If a generator reaches limit (min/ max) then fix it at that limit. Go to next iteration by dropping this generation from optimization. After the solution of this iteration (with a generator fixed at its limit), examine whether the constraints are still binding or not. Compare λ with the incremental cost of generator at limit. For example, a generator at max limit

$$\begin{cases} \left.\frac{\partial C_i}{\partial P_i}\right|_{P_i^{max}} > \lambda, \ it\ can\ be\ back\ off\ from\ limit \\ \left.\frac{\partial C_i}{\partial P_i}\right|_{P_i^{max}} < \lambda, \ it\ stays\ at\ limit \end{cases}$$

A generator at min limit

$$\begin{cases} \left.\frac{\partial C_i}{\partial P_i}\right|_{P_i^{min}} > \lambda, \ it\ stays\ at\ limit \\ \left.\frac{\partial C_i}{\partial P_i}\right|_{P_i^{min}} < \lambda, \ it\ can\ generate\ more \end{cases}$$